CHARACTERISTICS OF JAPANESE MATHEMATICS LESSONS

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Japanese mathematics lessons, especially for elementary grades, include a significant amount of problem solving. This instructional approach, called structured problem solving, is designed to create interest in mathematics and stimulate creative mathematical activity in the classroom through students' collaborative work. The lesson usually starts with students working individually to solve a problem using their own mathematical knowledge. After working with problems, students bring various approaches and solutions to classroom discussion. The teacher then leads students in a whole-class discussion in order to compare individual approaches and solutions. This whole-class activity provides students with opportunities to develop their mathematical abilities including conceptual and procedural understanding.

THE REFORM MOVEMENT IN TEACHING AND LEARNING MATHEMATICS

Teaching mathematics through lectures may be an easy instructional method for teachers. When students are passively listening to teachers, however, their opportunities to understand mathematical concepts and procedures are not maximized. Rather than just listening to teachers talk, students need to be actively involved in mathematics and to do mathematical activities (Brown, 1994).

In Japan, the major reform movement in teaching and learning mathematics occurred during the 1970s and 1980s (Takahashi, 2000). One of the major aspects of this reform movement was to shift from a traditional classroom that focuses on teachers' instruction, to a student-centered classroom that focuses on students' engagement in mathematical activities. During this reform movement, Japanese mathematics educators and teachers worked collaboratively to find ways to implement the ideas of reform mathematics teaching and learning by referring to various documents published in the U.S. Those references included the National Research Council's (1989) "Everybody Count: A Report to the Nation on the Future of Mathematics Education," and the National Council of Teachers of Mathematics' "Curriculum and Evaluation Standards for School Mathematics" (National Council of Teachers of Mathematics, 1989). A basic assumption of the reforms is that students can learn by constructing their own conceptions of mathematics (National Research Council, 1989). In other words, students are viewed as active constructors of knowledge, rather than passive recipients of it (Brown, 1994). Based on the TIMSS videotape classroom study (1997), Stigler and Hiebert argue that Japanese mathematics lessons better exemplify current U.S. reform ideas than do typical U.S. mathematics lessons (1999).

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Student-centered instruction using problem solving as a foundation

Although images of good practice might not be exactly the same among all Japanese educators, most of them would agree that Japanese school mathematics has been strongly influenced by the emphasis of problem solving as a good practical application of reform mathematics.

Stigler and Hiebert described Japanese mathematics lessons as "structured problem solving" by summarizing several characteristics of Japanese mathematics lessons.

> In Japan, teachers appear to take a less active role, allowing their students to invent their own procedures for solving problems. And those problems are quite demanding, both procedurally and conceptually. Teacher, however, carefully design and orchestrate lessons so that students are likely to use procedures that have been developed recently in class. An appropriate motto for Japanese teaching would be "structured problem solving" (J. Stigler & Hielbert, 1999, p. 27)

Similar characteristics of Japanese mathematics lessons were also reported in the proceedings of the U.S.-Japan Seminar of Mathematical Problem Solving (Jerry P. Becker & Miwa, 1987; Jerry P Becker, Silver, Kantowski, Travers, & Wilson, 1990).

Japanese structured problem solving was built on the firm foundation of emphasizing story problems in mathematics teaching and learning. Historically, Japanese mathematics teaching and learning has been focused on developing mathematical thinking skills by using a variety of story problems. In fact, teaching patterns similar to the ones that Becker reported (Jerry P Becker, Silver, Kantowski, Travers, & Wilson, 1990) were found not only widely throughout Japan but also in earlier documents. Those earlier documents include publications as early as 1937 (Minoru Yoshida, 1992). Based on the existing resources of story problems and of lesson plans focusing on promoting mathematical thinking, Japanese teachers, researchers, and administrators worked collaboratively through Lesson Study, a professional development approach that is popular in Japan, to develop mathematics instruction by referring to Polya's (1945) four phases of problem solving work (Takahashi, 2000). Studies of U.S. documents of mathematical problem solving that focused on teaching mathematical thinking skills (Lester & Garofalo, 1982; Schoenfeld, 1985) also influenced the Japanese mathematical reform movement.

Although Japanese structured problem solving has been influenced by U.S. research on problem solving, it is not the same as the problem solving approach used in the U.S. In the U.S., problem solving is often viewed as an approach to develop problem-solving skills and strategies. As a result, U.S. mathematics lessons employing the problem solving approach are usually focused on the process of solving a problem and not necessary focused on developing mathematical concepts and skills. These problem-solving lessons often end when each student comes up with a solution to the problem.

In Japan, on the other hand, problem solving is often viewed as a powerful approach for developing mathematical concepts and skills. Thus, Japanese teachers use problem solving not only for lessons that focus on developing problem-solving skills and strategies but also throughout the curriculum in order to develop mathematical concepts, skills, and procedures.

Structured problem solving

Structured problem solving is a major instructional approach in Japanese mathematics education. Structured problem solving is designed to achieve two extremely important goals that are essential to the Japanese mathematics reform curriculum: to create interest in mathematics, and to stimulate creative mathematical activity in the classroom during the collaborative work of students.

This instructional approach emphasizes the process of problem-solving activities and provides students with opportunities to re-invent mathematical ideas and concepts by themselves. This is why the lesson usually starts with students working individually to solve a problem using their own mathematical knowledge. After working with problems, students bring to classroom discussion several different approaches and solutions. The teacher then leads students in a whole-class discussion in order to compare individual approaches and solutions. This whole-class activity provides students with opportunities to develop their mathematical abilities including conceptual and procedural understanding (Figure 1).

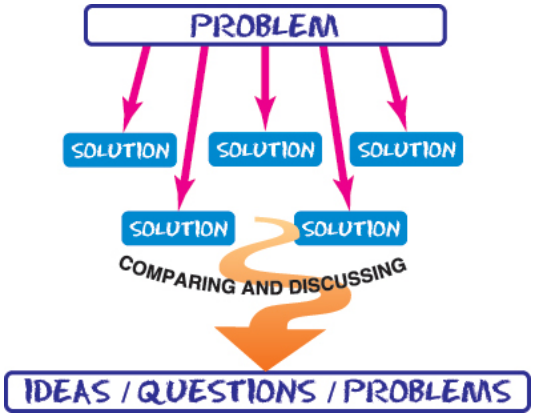

Figure 1: Structured Problem-Solving

Using structured problem solving as a major instructional approach, Japanese mathematics lessons bear notable characteristics (Jerry P Becker, Silver, Kantowski, Travers, & Wilson, 1990; Stevenson & Stigler, 1992; J. Stigler & Hielbert, 1999; J. W. Stigler, 1987; J. W. Stigler, Gonzales, Kawanaka, Knoll, & Serrano, 1999). Among these characteristics, the following three major characteristics will be discussed in this paper.

- Carefully selected word problems and activities, and their cohesiveness
- Extensive discussion (*Neriage*)
- Emphasis on blackboard practice (*Bansho*)

Carefully selected word problems and activities, and their cohesiveness

Typically, each Japanese mathematics lesson is designed around solving a single problem to achieve a single objective in a topic. In order for students to achieve the objective, the teacher carefully selects a problem an activity for the day. It is rarely seen that a lesson includes two or more problem-solving activities, although teachers might give students a couple of extended problem or exercises after a major problem solving. Thus, the selection of a problem for the problem-solving activity in each class is extremely critical for teachers when they plan a lesson.

When we look closely at Japanese mathematics textbooks, the use of carefully selected problems and activities and their cohesiveness emerge. The following examples are from one of the elementary grade textbook series[1] in Japan.

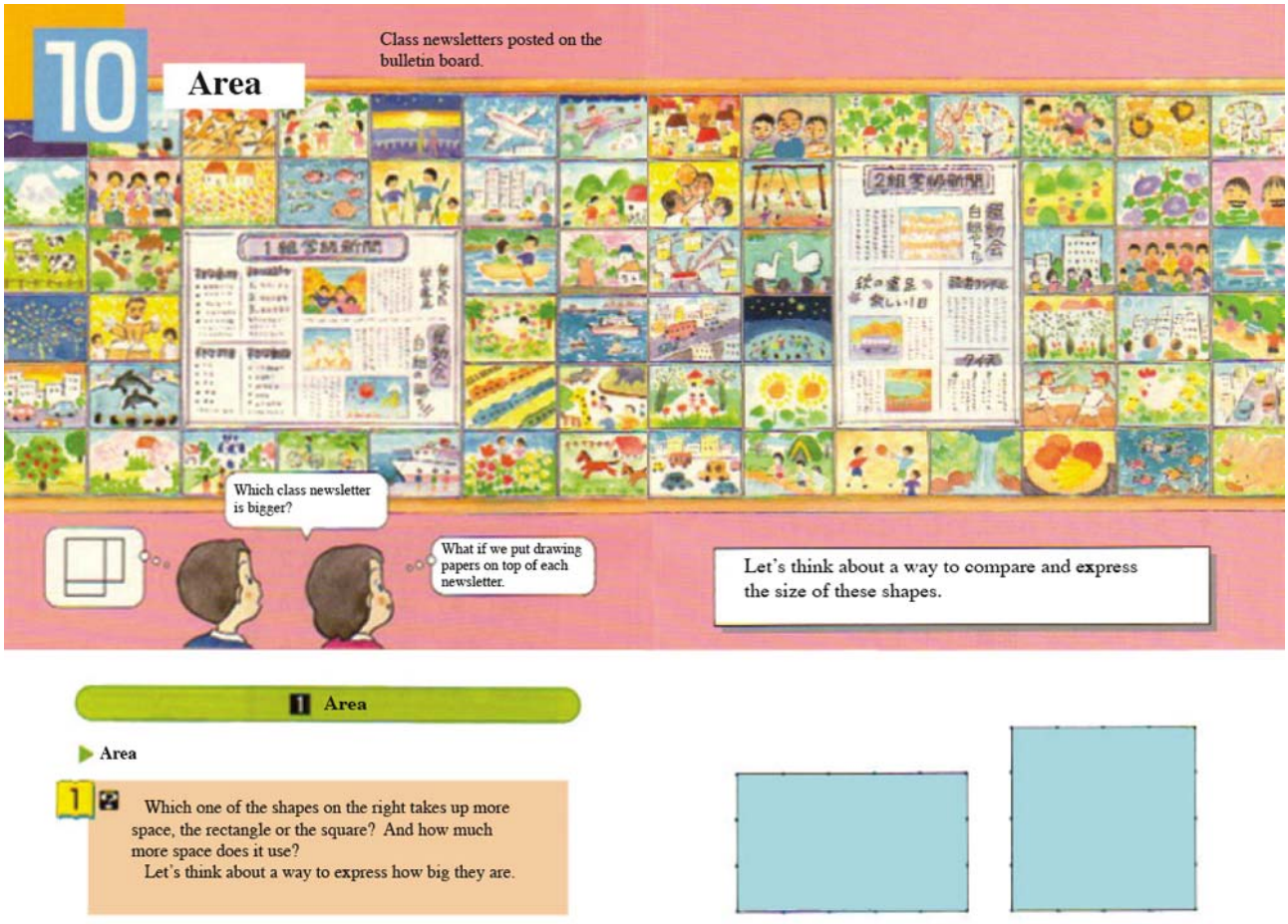

Figure 2: Japanese fourth grade mathematics textbook (4B) pp.22-23

To develop the concepts and skills for finding the area of basic figures, the Japanese textbook includes an introductory activity in the first grade. The page is designed to introduce the concept of direct comparison, indirect comparison, and comparison using

[1] The textbook series that is referred in this paper is the English translation of the textbook series for 1st grade to 6th grade published by the Tokyo Shoseki based on the 1989 National Course of Study translated by the Global Education Resources, NJ (www.globaledresources.com).

an arbitrary unit. The page is also designed to provide mathematical situations from students' daily life in order for them to become familiar with comparing area.

Building upon the activity in the first grade, the fourth grade textbook includes a unit designed for students to develop the concepts of finding the area of rectangles and squares by using multiplication formulas. This unit begins with a problem from students' familiar context. The rectangle and the square are given to students to compare which is bigger (Figure 2). The sizes of these two geometric figures are carefully chosen. For example, the perimeters of these two figures are the same but their areas are different by the area of a square with 1 cm sides. This is used because students tend to be confused by a typical misunderstanding that the areas of shapes can be determined by their perimeters. In order for students to overcome this misunderstanding, the textbook intentionally chose two shapes with the same perimeters but different area.

Through this problem-solving activity, students are expected to use their prior knowledge to compare the area of the rectangle and the square by using the area of square with 1 cm sides as an arbitrary unit. Then, measuring the areas of the rectangle and the square by using 1 cm^2 is introduced as measurement by a universal unit.

This unit also includes problem-solving activities to extend their capacity to use these formulas on irregular shapes (Figure 3). This problem-solving activity is designed to provide students with an opportunity to apply their previous learning, which is the formula for finding the area of a rectangle and a square, to a new situation. This experience will be the foundation for developing the formulas for finding the areas of other basic shapes in fifth grade.

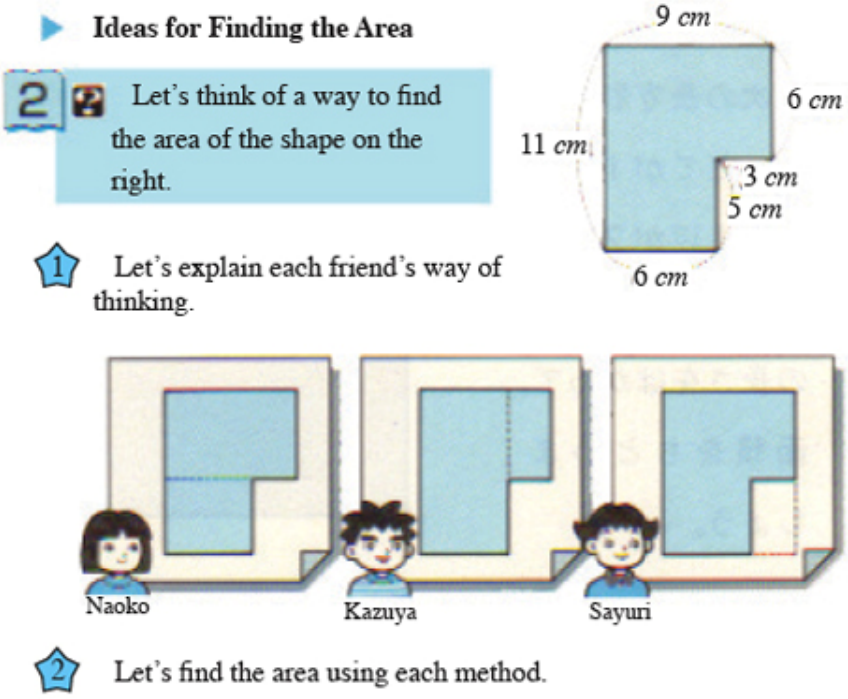

Figure 3: Japanese fourth grade mathematics textbook (4B) p.28

Building upon what they learned in the fourth grade, students in the fifth grade are given opportunities to develop the formulas for finding the area of a parallelogram, triangle, trapezoid, rhombus, regular polygon, and circle. Throughout the fifth grade students are expected to develop formulas for finding these basic figures through hands-on activities such as cutting and rearranging shapes on grid papers.

The sequential order of the problems and activities in each unit is carefully designed in order for students to develop their concepts and skills. A unit in Japanese mathematics textbooks is considered as a series of problems and activities rather than a set of problems and activities.

Extensive discussion (*Neriage*)

In the structured problem-solving approach, Japanese teachers emphasize that one of the most important roles of the teacher during a lesson is to facilitate mathematical discussion after each student comes up with a solution. When the teacher presents a problem to students without giving a procedure, it is natural that several different approaches to the solution will come from the students. Thus, the textbooks include examples of students' typical approaches and ideas. Because the goal of the structured problem-solving approach is to develop students' understanding of mathematical concepts and skills, a teacher is expected to facilitate mathematical discussion for students to achieve this goal. This discussion is often called *Neriage* in Japanese, which implies polishing ideas. In order to do this, teachers need a clear plan for the discussion as a part of their lesson plans, which will anticipate the variety of solution methods that their students might bring to the discussion. These anticipated solution methods will include not only the most efficient methods but also ones caused by students' misunderstandings. Thus, anticipating students' solution methods is a major part of lesson planning for Japanese teachers.

Towards the end of a lesson, a teacher often lead the lesson to pull all the different approaches and ideas together to see the connection. Then, he or she summarizes the lesson to help students achieve the objective of the lesson. The teacher often asks students to reflect on what they have learned during the lesson.

Emphasis on blackboard practice (*Bansho*)

Another notable characteristic of Japanese mathematics lessons is the use of mathematical expressions, figures, and diagrams on a large-size blackboard (Jerry P Becker, Silver, Kantowski, Travers, & Wilson, 1990; J. Stigler & Hielbert, 1999). Japanese classrooms are equipped with a large blackboard at the front. Yoshida (2005) summarizes how Japanese teachers use the blackboard during mathematics lessons as follows:

- To keep a record of the lesson
- To help students remember what they need to do and to think about
- To help students see the connection between different parts of the lesson and the progression of the lesson

- To compare, contrast, and discuss ideas that students present
- To help to organize student thinking and discovery of new ideas
- To foster organized student note-taking skills by modelling good organization

Discussion that compares and synthesizes several different solution methods demands

Figure 4: Use of blackboard (*Bansho*)

that students not only clarify the idea behind each method and justify the adequateness of each one. It is also important to examine the limitations of each method. To facilitate such discussion, Japanese teachers use the blackboard as a visual aid for students to participate in discussion throughout all the grades while considering student level of understanding of mathematics as well as their communication skills (Figure 4). Since this blackboard practice is an important skill for teachers to develop, Japanese teachers use a special term, *Bansho*, to discuss issues regarding the board writing skill. Thus, developing a plan for using the blackboard is another major component of lesson planning. In fact, some schools choose developing teachers' skills for using the blackboard effectively as a school goal when conducting school-based professional development.

CONCLUSION

Japanese mathematics teaching, especially for elementary grades, includes a significant amount of problem solving in order to provide students the environment to construct their understanding of concepts and procedures in mathematics. Although Japanese textbooks exemplify the characteristics of mathematics lessons, those characteristics do not solely came from the textbook authors' effort. It is important to note that the characteristics of Japanese mathematics lessons are the result of collaboration among Japanese educators, researchers, and policymakers, including efforts from the lesson study practitioners. Moreover, the effort to improve mathematics teaching and learning is still ongoing.

References

Becker, J. P., & Miwa, T. (1987). *Proceedings of the U.S.-Japan Seminar on Mathematical Problem Solving (Honolulu, Hawaii, July 14-18, 1986)* (COLLECTED WORKS - Conference Proceedings No. INT-8514988): Southern Illinois Univ., Carbondale.[JIM81075].

Becker, J. P., Silver, E. A., Kantowski, M. G., Travers, K. J., & Wilson, J. W. (1990). Some Observations of Mathematics Teaching in Japanese Elementary and Junior High Schools. *Arithmetic Teacher, 38*(2), 12-21.

Brown, A. (1994). The Advance of learning. *Educational Researcher, 23*(8), 4-12.

Lester, F. K., & Garofalo, J. (1982). *Mathematical problem solving: Issues in research*. Philadelphia: The Franklin Institute Press.

National Council of Teachers of Mathematics. (1989). *Curriculm and evaluation standards for school mathematics*. Reston, Virginia: National Council of Teachers of Mathematics.

National Research Council. (1989). *Everybody counts: A report to the nation on the future of mathematics education*. Washington, D. C.: National Academy Press.

Polya, G. (1945). *How to solve it: A new aspect of mathematical method*. Princeton, New Jersey: Princeton University Press.

Schoenfeld, A. H. (1985). *Mathematical problem solving*. Orlando, Florida: Academic Press Inc.

Stevenson, H., & Stigler, J. (1992). *The learning gap*. New York: Summit.

Stigler, J., & Hielbert, J. (1997). Understanding and imporving mathematics instruction: An overview of the TIMSS video study. *Phi Delta Kappan, 79*(1), 14-21.

Stigler, J., & Hielbert, J. (1999). *The teaching gap: Best ideas from the world's teachers for imporving education in the classroom*. New York: Free Press.

Stigler, J. W. (1987). Mathematics Classrooms in Japan, Taiwan, and the United States. *Child Development, 58*(5), 1272-1285.

Stigler, J. W., Gonzales, P., Kawanaka, T., Knoll, S., & Serrano, A. (1999). The TIMSS Videotape Classroom Study: methods and findings from and exploratory research project on eighth-grade mathematics instuction in Germany, Japan, and the United States. *Educational Statistics Quarterly, 1*(2), 109-112.

Takahashi, A. (2000). Current Trends and issues in lesson study in Japan and the United States. *Journal of Japan Society of Mathematical Education, 82*(12), 15-21.

Yoshida, M. (1992). Remarks on Japanese Mathematics Lessons. In T. Miwa (Ed.), *Nihon to America no Suugakuteki Mondaikaiketu no Shido (Teaching mathematical problem solving in Japan and the U.S.)* (pp. 188-221). Tokyo: Toyokan Publishing Company.

Yoshida, M. (2005). Using lesson study to develop effective blackboard practice. In P. Wang-Iverson & M. Yoshida (Eds.), *Building our understanding of lesson study* (pp. 93-100). Philadelphia: Research for Better Schools.